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Flat-top Laser Pulse Shaping in the UV with alpha-BBO (α-BBO) Birefringent Crystals

J. G. Power
AWA Group

Abstract.

A new method for making longitudinally flat-top laser pulses, based on stacking UV laser pulses with birefringent crystals, is presented. The purpose of this WF note is to make a preliminary analysis of the viability of the scheme.

1. Introduction.

It is well known that the uniform cylinder (UC) laser pulse shape (flat-top intensity distribution in both the transverse and longitudinal dimensions) produces a lower transverse emittance than the ordinary Gaussian laser pulse shape. While, the transverse profile is easily achieved by imaging an iris onto the photocathode, it has proven difficult to achieve a reasonably flat longitudinal profile. Two methods of longitudinal pulse shaping have been used for RF Photocathode guns so far. The first, *spectral masking*, is a frequency domain technique that manipulates light in the Fourier Transform plane to produce the flat-top pulse shape [1]. If this technique is applied in the IR, then pulse shape tends not to be preserved after harmonic generation. On the other hand, if the spectral masking is applied in the UV the insertion loses can be very large. The second, *temporal pulse stacking*, directly stacks longitudinally Gaussian pulses into an approximate flat-top and has so far been accomplished using a combination of half-wave plates and beamsplitting cube polarizers [2]. However, this system is difficult to adjust and align in practice. In this paper, we propose a new technique of *temporal pulse stacking* based on UV birefringent crystals [3, 4].

2. A Basic Description of Temporal Pulse Stacking.

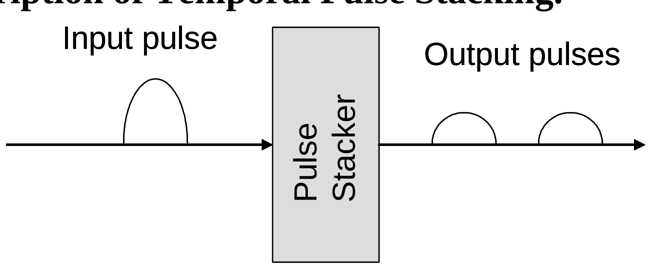

Figure 1) Generic Pulse Stacker. An input pulse is split in two and stacked, one after the other.

Laser pulse stacking [3] is the process of splitting a single laser pulse into a train, or stack, of *N* laser identical pulses separated in time (Figure 1). The name, pulse staking, is somewhat of a misnomer since it implies we already have a group of pulses to stack; a more appropriate name might be pulse splitting and stacking.

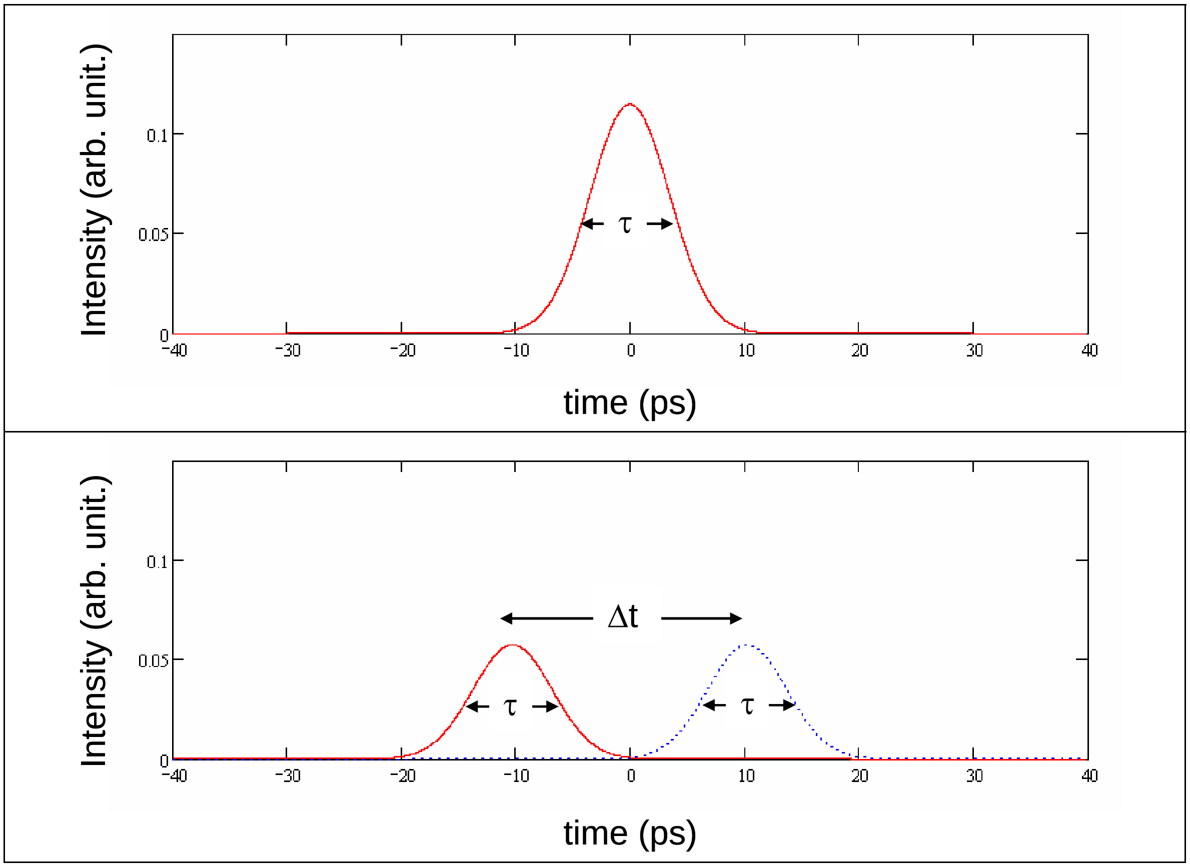

Figure 2) The Gaussian input pulse is shown on top with a FWHM, τ = 8.173 ps. The output pulses are separated by a time Δt and are duplicates of the input pulse except at half the intensity.

For a specific example of pulse stacking consider Figure 2 and the following discussion. Start with a single Gaussian input pulse defined by,

$$f(t,\sigma,\mu) := \frac{1}{\sigma\cdot\sqrt{2\cdot\pi}}\cdot\exp\left[\frac{-(t-\mu)^2}{2\cdot\sigma^2}\right] \qquad \textbf{(1)}$$

where the pulse duration FWHM = $\tau = 2\sqrt{2\ln 2}\sigma \approx 2.35\sigma$ and μ = mean offset from zero and σ is the rms standard deviation of the pulse. A pulse stacker is then designed to produce two Gaussian pulses separated in time by Δt. If the separation time, Δt, can be controlled then the output pulses can be stacked into the desired longitudinally flat-top pulse as is discussed below.

3. Temporal Walk-Off in Birefringent Crystals.

Birefringent crystals [5] are anisotropic materials that have different indices of refraction for the two difference polarizations of the incident light [3]. The ray that is polarized perpendicular to the optical axis of the crystal (or the axis of anisotropy) is called the *ordinary (o) ray* and the ray that is polarized parallel to the optical axis is called *extraordinary (e) ray*. The magnitude of the birefringence of the crystal is difference between the two indices,

$$\Delta n = n_e - n_o \tag{2}$$

where n_o and n_e are the **index of refraction** of the *o ray* and the *e ray*. (Note that the explicit dependence on n = n (λ) is dropped throughout the paper). Empirical formulas for $n_e(\lambda)$ and $n_o(\lambda)$ are available in the literature and I list two of them here for later use. For α-BBO the Sellmeier equations (λ in μm) are [8],

$$\begin{aligned} \mathrm{no\alpha BBO}(\lambda) &:= \sqrt{2.7471 + \frac{0.01878}{\lambda^2 - 0.01822} - 0.01354\lambda^2} \\ \mathrm{ne\alpha BBO}(\lambda) &:= \sqrt{2.3174 + \frac{0.01224}{\lambda^2 - 0.01667} - 0.01516\lambda^2} \end{aligned} \tag{3}$$

and the Laurent series equation for Crystal Quartz [9],

$$\begin{aligned} \mathrm{noSiO2}(\lambda) &:= \sqrt{2.35728 - 0.0117 \cdot \lambda^2 + \frac{0.01054}{\lambda^2} + \frac{1.34143 \cdot 10^{-4}}{\lambda^4} + \frac{-4.45368 \cdot 10^{-6}}{\lambda^6} + \frac{5.92362 \cdot 10^{-8}}{\lambda^8}} \\ \mathrm{neSiO2}(\lambda) &:= \sqrt{2.3849 - 0.01259 \cdot \lambda^2 + \frac{0.01079}{\lambda^2} + \frac{1.65181 \cdot 10^{-4}}{\lambda^4} + \frac{-1.94741 \cdot 10^{-6}}{\lambda^6} + \frac{9.36476 \cdot 10^{-8}}{\lambda^8}} \end{aligned} \tag{4}$$

Temporal walk-off [6] is the term used to describe the temporal separation (Δt) that develops as the *o ray* and *e ray* propagate through the crystal. This loss of temporal overlap is due to the difference in the group index of refraction seen by the two rays. A short laser pulse (high bandwidth) incident on a birefringent crystal will be split into two pulses separated in time due to the differences in group velocity. The separation is the product of the crystal length, *L*, and the group velocity mismatch (*GVM*),

$$\Delta t = L\left(\frac{1}{v_{ge}} - \frac{1}{v_{g0}}\right) \tag{5}$$

where $v_{go}(=c/n_{go})$ and $v_{ge}=(c/n_{ge})$ are the group velocities for the *o ray* and *e ray* respectively, c is the speed of light in vacuum, and n_{ge} and n_{go} are **group index of refraction** for the *e ray* and *o ray* respectively. (Note that this is not the same as values given in Eqn. 2.) The group index n_g is [7],

$$n_g = n - n\left(\frac{dn}{d\lambda}\right) \qquad (6)$$

and can be calculated from Equations (3) and (4). Note that positive birefringence occurs when the group velocity of the *o ray* greater than that of the *e ray* ($v_{go}>v_{ge}$). This is equivalent to extraordinary index greater than the ordinary ($n_{ge} > n_{go}$). Using these definitions we can rewrite equation 5 as,

$$\Delta t = L\frac{\Delta n_g}{c} \qquad (7)$$

where $\Delta n_g = n_{ge} - n_{go}$ is the group birefringence. Comparing Equations (5) and (7) we can write the group velocity mismatch as, $GVM = \Delta n_g / c$, and finally arrive at the desired result for the temporal delay,

$$\Delta t = L * GVM \qquad (8)$$

Note that the GVM can be thought of as the normalized delay, or the delay per unit length with units of ps/mm, for example.

4. Birefringent Crystals in the UV.

The ideal crystal for the pulse stacking application should be strongly birefringent (large Δn) and transparent at the wavelength of interest. Pulse stacking based on birefringent crystals was first demonstrated [1] using a calcite crystal and a frequency-doubled Nd:YAG laser (λ = 532 nm). In that experiment, the crystal had a large group birefringence, Δn_g = 0.23. Using Equation 5, we calculate GVM = 0.76 ps/mm for calcite.

Table 1) Commercially Available uniaxial UV-transparent Birefringent Crystals

Crystal	Type*	λ (nm)	n_e/n_{ge}	n_o/n_{go}	Transmission (uncoated)	Δn_g	GVM (ps/mm)
Crystal Quartz[5]	Positive uniaxial	248.0	1.6129/ 1.7769	1.5964/ 1.723	~90% at 10 mm thick	0.054	0.180
α-BBO[4]	Negative uniaxial	248.0	1.6092/ 1.8425	1.7833/ 2.1295	~85% at 8.5 mm thick	-0.287	-0.957

*Positive uniaxial crystal has ne > no or, equivalently, $v_{go}>v_{ge}$.

The difficulty with temporal pulse stacking technique in the UV is due to the shortage of transparent and strongly birefringent crystals. While calcite's group birefringence is sufficiently strong, it is not transparent in the UV. Fortunately, an examination of commercially available uniaxial crystals (Table 1) reveals two promising candidates. The values for n_e and n_o (Table 1) were provided by the manufacturer, while the values for n_{ge}, n_{go}, Δn_g, and GVM were calculated from the equations in the previous section. From Table 1, crystal quartz appears to be slightly more transparent, but α-BBO's large GVM, nearly 1 ps/mm, makes it a better choice for out application.

5. Temporal Pulse Stacking.

In this section we show how to create the desired longitudinally flat-top laser pulse shape with birefringent crystals. The basic idea is to use a series of birefringent crystals to divide the input pulse into a stack of Gaussians and then recombine them. We begin by describing in detail the specific case of 4 stacked Gaussians. We end by showing more general results.

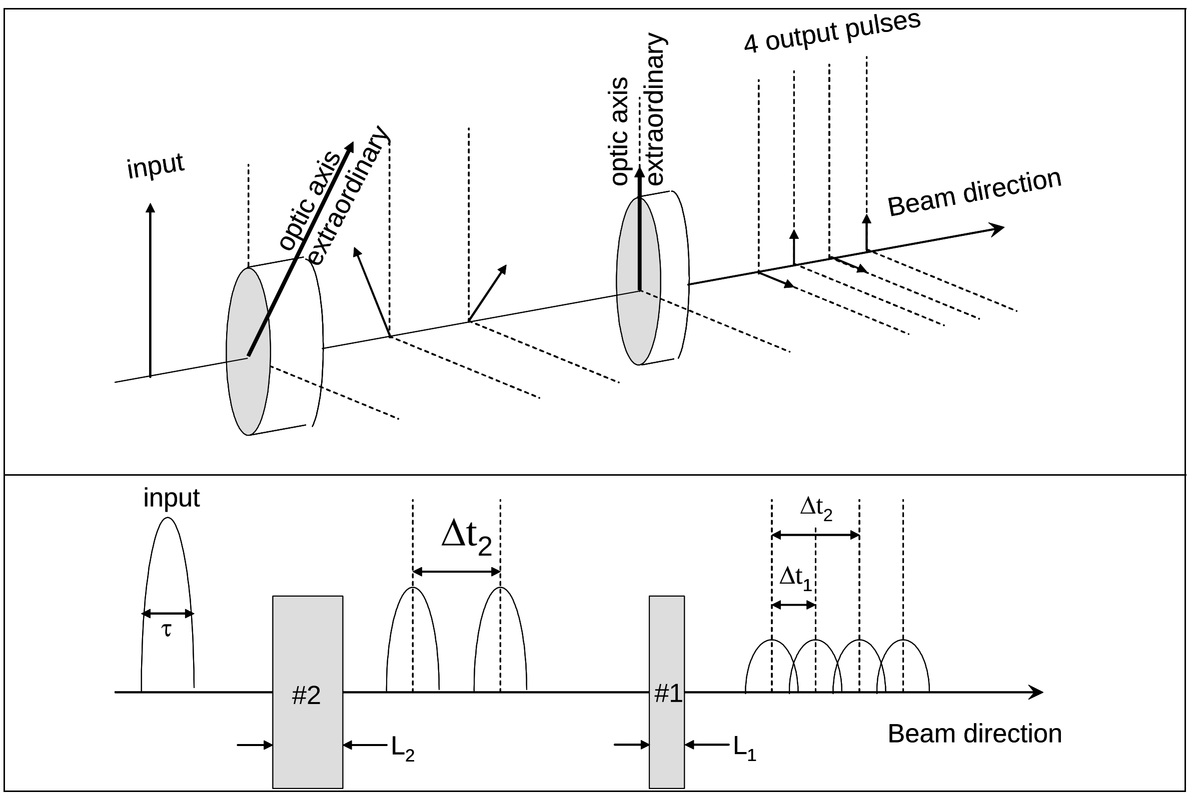

Figure 3) Schematic diagram showing 2 birefringent crystals used to produce 4 stacked Gaussians. **Top**: The optic axis of the first crystal is oriented at a 45^0 angle relative to the input polarization while the optic axis of the second crystal is oriented in the same direction as the input polarization. **Bottom:**. The input pulse is split into two intermediate pulses, separated by Δt_2, after the crystal of length L_2. The intermediate pulses are further split and delayed by Δt_1 by the crystal of length L_1 to produce the 4 output pulses.

A sketch of how to use 2 birefringent crystals to create 4 stacked Gaussian pulses, approximating the flat-top, is shown in Figure 1. The input Gaussian pulse is linearly polarized in the vertical direction while the optic axis of crystal #2, of length L_2, is tilted at a 45^0 angle relative to the vertical. For α-BBO (a negative uniaxial crystal), the *e-ray* (or the component of the input pulse that is parallel to the optic axis), will move ahead of *o-ray* the component of the input pulse that is perpendicular to the optic axis by the amount, Δt_2 (Eqn 8) with crystal length L_2. In this case ($n_e < n_o$), the extraordinary axis is the fast axis. The 45^0 orientation creates equal intensity *e-ray* and *o-ray*. (Notice that the relative intensity between the two rays can be controlled by a simple rotation of the optic axis which leaves open the possibility for ramped pulse generation.) The two (intermediate) pulses emerging from crystal #2 are now themselves oriented at 45^0 to the vertical. The next crystal (crystal #1) has its optic axis oriented in the same direction as the input pulse; i.e. in the vertical. When the two intermediate pulses pass through crystal #1, they are each further divided into 2 more pulses separated by Δt_1 (Eqn 8) with crystal length L_1, thus producing the 4 output pulses.

In the example just considered, an input pulse of intensity I_o, incident on a pulse stacker of 2 crystals, will produce 4 output pulses of intensity $I_o/4$; less insertion losses. In general, it is easy to see that N crystals will generate 2^N output pulses of intensity I_o/N; less insertion losses. Therefore, the number of pulses in the stack is limited by the transparency of the crystals and the amount of laser energy needed for the application.

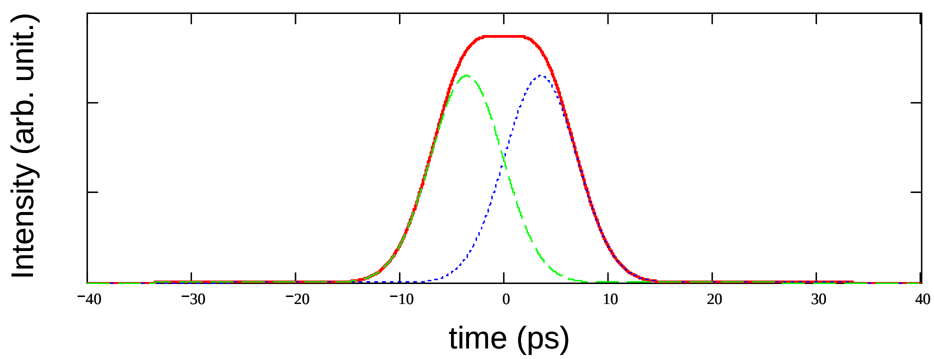

Figure 4) Resultant flat-top for L1 = 7.393 mm and FWHM = 15.24 ps

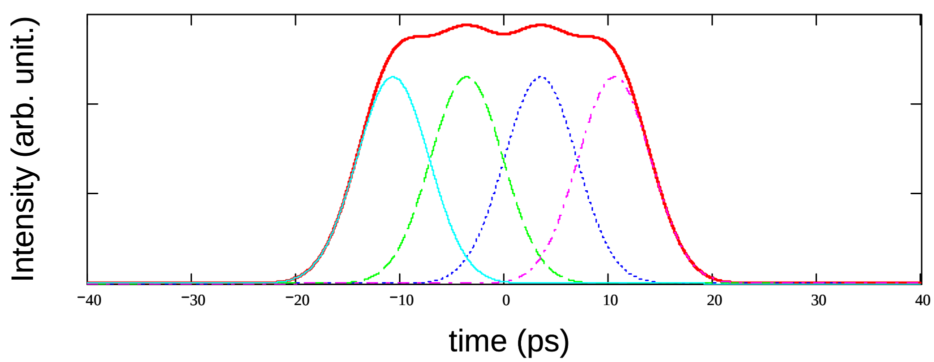

Figure 5) Resultant flat-top for L1 = 7.393 mm, L2=14.786 mm, and FWHM = 29.40 ps

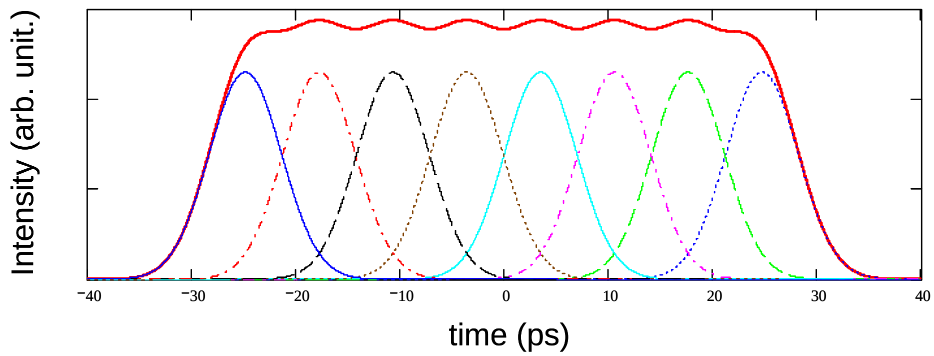

Figure 6) Resultant flat-top for L1 = 7.393 mm, L2=14.786 mm, and L3=29.572 mm; FWHM = 57.70 ps

We now briefly consider 3 (non-optimized) pulse stacking cases for use at the AWA. We will use an a-cut, α-BBO crystal with GVM= -0.957 ps/mm and the nominal AWA laser pulse with λ = 248 nm, τ_{FWHM} = 8.16 ps, and rise time (10%-90%) =5.86 ps.

(Case 1) A single crystal of length: L1 = 7.393 mm produces two Gaussian pulses with their peaks separated by $\Delta t = 7.077\,ps$. The net pulse length (Fig. 4) for the 2-gaussian case is $\tau 2_{FWHM} = \tau_{FWHM} + |\Delta t|$ = 15.24 ps.

(Case 2) Two crystals of length: L1 = 7.393 mm and L2 = 2*L1 = 14.786 mm. $\tau 4_{FWHM} = \tau_{FWHM} + 3 * |\Delta t|$ = 29.4 ps.

(Case 3) Three crystals of length: L1 = 7.393 mm, L2 = 2*L1 = 14.786 mm, and L3 = 4*L1 = 29.572 mm $\tau 4_{FWHM} = \tau_{FWHM} + 7 * |\Delta t|$ = 57.70 ps.

5. TStep simulation for stacked Gaussian Pulses.

In this section, TStep simulations are made for the 3 cases considered above and they are benchmarked to the nominal AWA case of a single Gaussian laser.

Table 2) TStep simulation results for the stacked pulses. Rise time(10%-90%) =5.86 ps

Case #	Number of Gaussian	FWHM (ps)	Phase	Emittance
Baseline	1	8.16	30.456	4.103
1	2	15.24	32.496	3.385

2	4	29.40	36.472	2.451
3	8	57.70	36.472	2.018

It is well known that the uniform cylinder (UC) laser pulse shape produces lower transverse emittance than Gaussian pulse shapes. In this section, we present the results of TStep simulations of the 3 cases considered above. The Matlab Optimization Toolbox was used to minimize the normalized rms x-emittance by varying the gun launch phase; the results are shown in Table 2 and Figure 7.

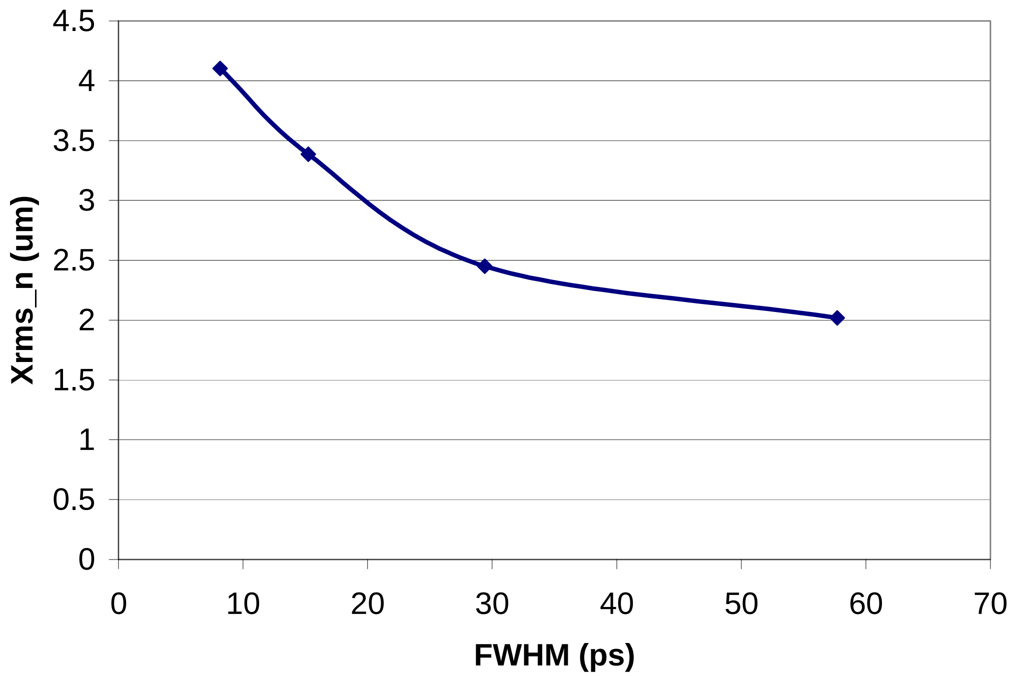

Figure 7) The normalized rms projected X emittance vs. laser pulse length.

The input file used for the TStep simulation of the baseline case is listed in its entirety in Appendix A. The only difference between the input files amoung the 4 cases is the value of **Z0** on the **run card** and the **longitudinal parameters** on the **input card(s)**. The differences are listed Appendix B. Lastly, the Matlab minimization routine and its output are shown in Appendix C.

Future Directions

Preliminary analysis of the birefringent crystal scheme for directly stacking UV laser pulse presented in this paper looks promising. The next step will be to obtain and characterize a long α-BBO crystal. The delay between the *e-ray* and the *o-ray* (temporal walk-off) will be measured with a streak camera and the attenuation with an energy meter. The 25 mm α-BBO crystal should create a delay of 23.925 ps.

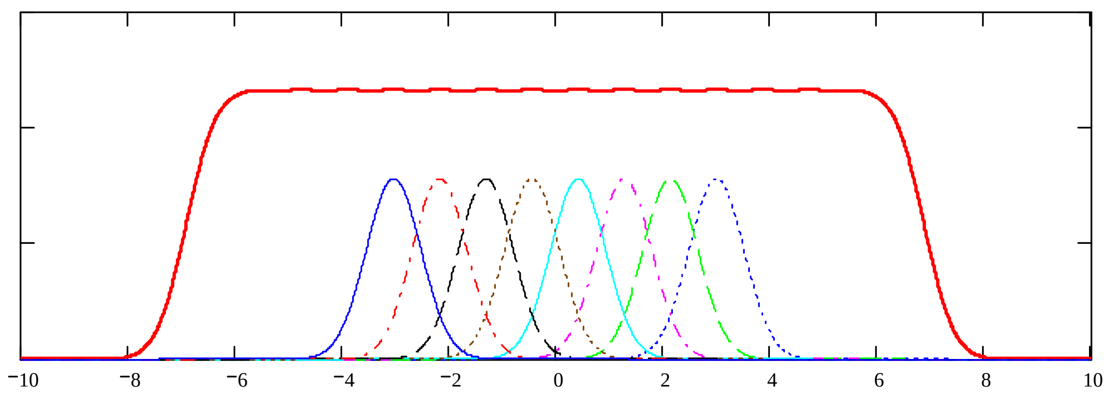

Figure 8) Resultant flat-top for L1 = 0.9 mm, L2=1.8 mm, L3=2.7 mm, and L4=3.6 mm; FWHM = 14.12 ps and rise-time = 0.78. Fundamental Gaussian, rms=1.2 ps.

In the near future we will retune the stretcher in the AWA laser system to see if shorter pulses can be generated. If it can be made shorter, then we can create a high quality flat-top pulse. For example, using a Gaussian with FWHM = 1.2 ps and 4 crystals to stack 16 pulses a flat-top pulse rise-time-0.78 ps and FWHM=14.12 ps can be created. (Fig. 8)

Conclusion

In this WF note, a new method for temporal laser pulse stacking based on birefringent crystals was presented. It was shown that N crystals can be stacked together to create 2^N stacked pulse to approximate a flat-top pulse train. High quality pulse trains can be created if a short laser pulse is available.

References

[1] A.M. Weiner, "Femtosecond pulse shaping using spatial light modulators", Rev. Sci. Inst. 71[5], 1929 (May 2000).

[2] J. Li, et al., NIM A 564, p. 57 (2006).

[3] Picosecond pulse stacking in calcite
Harry E. Bates, Robert R. Alfano, and Norman Schiller

[4] Shian Zhou and Frank W. Wise, Dimitre G. Ouzounov April 1, 2007 / Vol. 32, No. 7 / OPTICS LETTERS

[5] http://en.wikipedia.org/wiki/Birefringence

[6] http://www.rp-photonics.com/temporal_walk_off.html

[7] http://en.wikipedia.org/wiki/Refractive_index

[8] http://www.newlightphotonics.com/alpha-BBO.html

[9] http://www.cvilaser.com/Common/PDFs/Dispersion_Equations.pdf

[10] http://www.u-oplaz.com/crystals/crystals30.htm

Appendix A

TStep INPUT file

```
!Test Stand Beamline
!Laser sigPhi = 3.478 psec = 1.626 degree; (using phimax=6.5 degree)
!Gun normalization 47.05 -> 100 MV/m (36.2285 -> 77 MV/m) (32.935->70MV/m)
!Linac normalization (dE = E0*T*sin(phi); T=0.78 from SF ????? DW)
!Z0=-0.0008246 for phimax= 6.5 deg;
!Z0=-0.0010337 for phimax = 8.156 degree (two stacked gaussian, dT=
!

run 1 1 1300.0 -0.001034 1.e-6 7
title
  AWA Test Stand (36.2285 -> 77 MV/m)

!INPUT 9, N,  sigR, Rmax,  sigPhi,  Phimax dphi Nb1 Nb2
input    9  1000  20.     0.2     1.626        6.5        0     3    5

!gun begin
drift 0 2.5 1
cell 0.100000 2.500000 1 32.496278 36.228500 1 1 -1
cfield 1
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0001.T7
cell 0.150000 2.500000 1 32.496278 36.228500 2 1 -1
cfield 2
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0025.T7
cell 0.250000 2.500000 1 32.496278 36.228500 3 1 -1
cfield 3
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0050.T7
cell 0.500000 2.500000 1 32.496278 36.228500 4 1 -1
cfield 4
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0100.T7
cell 0.750000 2.500000 1 32.496278 36.228500 5 1 -1
cfield 5
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0175.T7
cell 1.250000 2.500000 1 32.496278 36.228500 6 1 -1
cfield 6
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0300.T7
cell 2.000000 2.500000 1 32.496278 36.228500 7 1 -1
cfield 7
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0500.T7
cell 2.000000 2.500000 1 32.496278 36.228500 8 1 -1
cfield 8
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0700.T7
cell 2.000000 2.500000 1 32.496278 36.228500 9 1 -1
cfield 9
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun0900.T7
cell 2.000000 2.500000 1 32.496278 36.228500 10 1 -1
cfield 10
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun1100.T7
cell 2.000000 2.500000 1 32.496278 36.228500 11 1 -1
cfield 11
```

```
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun1300.T7
cell 2.000000 2.500000 1 32.496278 36.228500 12 1 -1
cfield 12
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun1500.T7
cell 2.000000 2.500000 1 32.496278 36.228500 13 1 -1
cfield 13
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun1700.T7
cell 2.000000 2.500000 1 32.496278 36.228500 14 1 -1
cfield 14
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun1900.T7
cell 2.000000 2.500000 1 32.496278 36.228500 15 1 -1
cfield 15
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun2100.T7
cell 4.000000 2.500000 1 32.496278 36.228500 16 1 -1
cfield 16
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun2500.T7
cell 4.300000 2.500000 1 32.496278 36.228500 17 1 -1
cfield 17
D:\Argonne\PhysCodes\Parm\ParmFiles\TS\gun\fields\gun2930.T7
!gun end
!end of gun is at z=29.30 cm

drift  0.70 5.08 1  !end of this drift is z=30 cm
drift  1.00 5.08 1
drift  2.00 5.08 1
drift  2.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !end of this drift is z=50 cm
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !end of this drift is z=100 cm
drift  2.00 5.08 1  !end of this drift is YAG-1 (z=102cm)
drift  3.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  3.50 5.08 1  !end of this drift is entrance to linac (z=128.50 cm)
drift  1.50 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !z=150cm
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
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drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !z=200cm
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !z=250cm
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  3.50 5.08 1  ! 2nd YAG 278.5cm
drift  1.50 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !z=300cm
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1
drift  5.00 5.08 1  !z=350cm
drift  5.00 5.08 1
drift  2.00 5.08 1  !3rd YAG 357 cm (pepper pot)



!poisson -33.0, 1.00 0.0, 0.0, 0.0, 0.0  ! 40.49 + 0.35
!D:\Argonne\PhysCodes\Parm\ParmFiles\TS\solenoid\oldfields\a240b220.po7
!D:\Argonne\PhysCodes\Parm\ParmFiles\scratch\BFmax_Mscaled.T7

poisson 0.0, 1.00 0.0, 0.0, 0.0, 0.0  ! comments?
BFscaled_Mscaled.T7
zlimit 1.5

!bucking & focusing, always run at max
!bucking max_I=553 Amps & Nturns=48
!focusing max_I=562 Amps & Nturns=48 (July 25, 2006)
!compromise: Use I=550 Amps, ---> maxNI= 26400 Amp*Turns

!matching, scan until we have a good spot
!matching max_i=440 Amps & nturns=76    (July 25, 2006)
```

```
!max = 440*76 = 33440 Amp*Turns

!a205b250.po7     A240B220.PO7

zout                                        !use this when running in batch mode
!zout 200 10 0 300 0 0  1 0  !use this when I want RFfld output from parmela, good to check field
normalization

output 5 !needed to use Pargraf

!SCHEFF            IT(amps),  RSC,  ZSC, NR, NZ, NBunch, LBunch, RingOption, MeshFactor, RWall, b
scheff                     1.30       0.4      2.0    30   900       0              0            9                  1.1        0.0   0.0

!START Phi   dPhi   nPhi spaceChargeSteps   outputSteps       !if spaceChrageSteps=0 --> S.C. off
start  0.0   0.1    200        1           10  ! 20 degrees  (this is about zref=0.36 cm from cathode)

scheff  1.30            0.6  1.5   36 300     0      0        9          1.1        0.0  0.0
continue     0.1    500        1           10  ! 70 degrees totally (this is about zref=3.3 cm from cathode)

scheff  1.30            1.0  1.5   36 300     0      0        9          1.1        0.0  0.0
continue     1.0    100        1           100  ! 170 degrees totally (this is about zref=9.6 cm from cathode)
continue     2.0    100        1           100  ! 370 degrees totally (this is about zref=22.4 cm from cathode)
continue     4.0    100        1           100  ! 770 degrees totally (this is about zref=48 cm from cathode)
continue    10.0  10000        1           100  ! +10000
continue    20.0  20000        1           100  ! +20000


!save 2
!scheff 130.0 1.2 4.5 35 320 0 0 3 1.5 0.0 0.0
!restart 1 100000 1 50 0 2


end
```

Appendix B: Individual TStep settings

The input file for the 4 cases was the same, except for the value of **Z0** in the **run** card and the **input 9** cards.

(1) Baseline: 1 gaussian (sigmaPhi=3.478 psec)

```
run 1 1 1300.0 -0.0008246 1.e-6 7
title
  AWA Test Stand (36.2285 -> 77 MV/m)

!INPUT 9, N,  sigR, Rmax,  sigPhi,  Phimax dphi Nb1 Nb2
input    9  1000  20.     0.2     1.626       6.5       0     3    5
```

(2) Case 1: two stacked Gaussian

```
run 1 1 1300.0 -0.001034 1.e-6 7
title
  AWA Test Stand (36.2285 -> 77 MV/m)

!INPUT          9, N,  sigR, Rmax,  sigPhi,  Phimax dphi Nb1 Nb2
input  9  1000  20.  0.2     1.626     6.5    1.656   3   5
input  9  1000  20.  0.2     1.626     6.5   -1.656   3   5
```

(3) Case 2: 4 stacked Gaussian

```
run 1 1 1300.0 -0.001444 1.e-6 7
title
  AWA Test Stand (36.2285 -> 77 MV/m)

!INPUT          9, N,  sigR, Rmax,  sigPhi,  Phimax    dphi  Nb1 Nb2
input  9  1000  20.  0.2     1.626     6.5    4.968   3   5
input  9  1000  20.  0.2     1.626     6.5    1.656   3   5
input  9  1000  20.  0.2     1.626     6.5   -1.656   3   5
input  9  1000  20.  0.2     1.626     6.5   -4.968   3   5
```

(4) Case 3: 8 stacked pulses

```
run 1 1 1300.0 -0.00231 1.e-6 7
title
```

AWA Test Stand (36.2285 -> 77 MV/m)

```
!INPUT          9, N,  sigR, Rmax,  sigPhi,  Phimax    dphi  Nb1 Nb2
input  9   500  20.  0.2     1.626     6.5   11.592   3   5
input  9   500  20.  0.2     1.626     6.5    8.280   3   5
input  9   500  20.  0.2     1.626     6.5    4.968   3   5
input  9   500  20.  0.2     1.626     6.5    1.656   3   5
input  9   500  20.  0.2     1.626     6.5   -1.656   3   5
input  9   500  20.  0.2     1.626     6.5   -4.968   3   5
input  9   500  20.  0.2     1.626     6.5   -8.280   3   5
input  9   500  20.  0.2     1.626     6.5  -11.592   3   5
```

Appendix C: Matlab m-file optiLaunch.m

This is the Matlab file used to optimize the emittance vs. phase

```
optfminsearch = optimset('TolFun', 0.010, 'TolX', 10.0)
%optfmincon = optimset('DiffMinChange', 0.5, 'DiffMaxChange', 10
,'TolFun', 0.01)


X0=35;lb=20; ub=60;

tic;
[x,fval,exitflag,output]=fminsearchbnd(@emit,X0,lb,ub,optfminsearch);
%[x, fval,exitFlag2,output2]=fmincon('emit', X0, [],[], [], [],lb,ub,
[],  optfmincon);
toc;
[x,fval]
```

Below are the outputs for the 4 cases:

(1) Baseline: 1 Gaussian

options =
TolFun: 0.01000000000000
TolX: 10
DiffMaxChange: 10
DiffMinChange: 0.01000000000000

50.00000000000000	6.82785397140716
55.20811931200062	7.02718880129641
43.64471050984295	6.18268720762860
36.87131069919539	4.91899495666649
25.13710349045213	5.31258702206932
30.45682479480787	4.10326835002520
25.13710349045217	5.30400245039224
33.57121069393679	4.28822714118085
27.61812101380337	4.57118973551897
31.98502333793721	4.11171976923524

Elapsed time is 1363.075889 seconds = **22 minutes**
30.45682479480787   4.10326835002520

(2) Case 1: two stacked Gaussian

optfminsearch =

TolFun: 0.01000000000000
TolX: 10

```
34.99999999999999   3.55139359413989
40.97651154205000   4.32588112948196
29.47464210703781   3.60337533295234
32.14825736243428   3.38937465570597
29.47464210703781   3.60337533295234
33.55582835460911   3.43450997053689
30.78528153706030   3.45719830134979
32.84696270910226   3.39995416542964
31.46070476785738   3.41669284365995
32.49627761521339   3.38573998604449
```

Elapsed time is 2682.636391 seconds. = **45 minutes**
32.49627761521339   3.38573998604449

(3) Case 2: 4 stacked Gaussian)

optfminsearch =
TolFun: 0.01000000000000
TolX: 10

```
34.99999999999999   2.46973763304829
40.97651154205000   2.68130534180654
29.47464210703781   3.07162260681840
37.96517438793728   2.49605660501122
32.14825736243428   2.69099384244904
36.47256990604419   2.45105596612325
37.96517438793726   2.49605660501122
35.73325467330833   2.45288483263150
```

Elapsed time is 4672.504742 seconds **= 77 minutes**
36.47256990604419   2.45105596612325

(4) Case 3: 8 stacked pulses

optfminsearch =
TolFun: 0.01000000000000
TolX: 10

```
34.99999999999999   2.04181082351947
40.97651154205000   2.58544545978363
29.47464210703781   2.78797279789873
37.96517438793728   2.09672083881710
32.14825736243428   2.32349714290291
```

36.47256990604419	2.01858712324269
37.96517438793726	2.09672083881710
35.73325467330833	2.02010584283933

Elapsed time is 4872.951418 seconds. = **81 minutes**
36.47256990604419   2.01858712324269